



Mathematical Modeling

Applications with GeoGebra™

Jonas Hall
Thomas Lingfjärd



WILEY

MATHEMATICAL MODELING

MATHEMATICAL MODELING

Applications with GeoGebra™

JONAS HALL AND THOMAS LINGEFJÄRD

WILEY

Copyright © 2017 by John Wiley & Sons, Inc. All rights reserved

Published by John Wiley & Sons, Inc., Hoboken, New Jersey

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permissions>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data

Names: Hall, Jonas, 1963– | Lingefj rd, Thomas, 1952–

Title: Mathematical modeling : applications with GeoGebra / Jonas Hall, Thomas Lingefj rd.

Description: Hoboken, New Jersey : John Wiley & Sons, Inc., [2017] | Includes index.

Identifiers: LCCN 2016009974 (print) | LCCN 2016019552 (ebook) | ISBN 9781119102724 (cloth) | ISBN 9781119102694 (pdf) | ISBN 9781119102847 (epub)

Subjects: LCSH: Mathematical analysis–Data processing. | Mathematical models–Data processing.

Classification: LCC QA300 .H353 2017 (print) | LCC QA300 (ebook) | DDC 003–dc23

LC record available at <https://lccn.loc.gov/2016009974>

Set in 10/12pt Times by SPi Global, Pondicherry, India

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CONTENTS

Preface	xi
Introduction	xiii
About the Companion Website	xxx
1 Some Introductory Problems	1
1.1 Ticket Prices, 3	
1.2 How Long Will the Pasture Last in a Field?, 7	
1.3 A Bit of Chemistry, 10	
1.4 Sydney Harbor Bridge, 16	
1.5 Perspective, 19	
1.6 Lake Erie's Area, 21	
1.7 Zebra Crossing, 25	
1.8 The Security Case, 31	
1.9 Personal Measurements, 34	
1.10 Height of the Body, 34	
1.11 Lamp Pole, 35	
1.12 The Skyscraper, 35	
1.13 The Fence, 35	
1.14 The Corridor, 35	
1.15 Bird Feeders, 35	
1.16 Golf, 36	
2 Linear Models	37
2.1 Are Women Faster Than Men?, 38	
2.2 Taxi Companies, 40	

- 2.3 Crime Development, 47
- 2.4 The Metal Wire, 52
- 2.5 Options Trading, 57
- 2.6 Flying Foxes, 62
- 2.7 Knots on a Rope, 66
- 2.8 The Candle, 66
- 2.9 Hooke's Law, 66
- 2.10 Ranking, 67
- 2.11 Dolbear's Law, 67
- 2.12 Man at Office, 68
- 2.13 A Stack of Paper, 68
- 2.14 Milk Production in Cows, 69

3 Nonlinear Empirical Models I 70

- 3.1 Galaxy Rotation, 71
- 3.2 Olympic Pole Vaulting, 73
- 3.3 Kepler's Third Law, 79
- 3.4 Density, 83
- 3.5 Yeast, 87
- 3.6 Cooling I, 88
- 3.7 Modeling the Population of Ireland, 93
- 3.8 The Rule of 72, 96
- 3.9 The Fish Farm I, 100
- 3.10 New Orleans Temperatures, 104
- 3.11 The Record Mile, 107
- 3.12 The Rocket, 107
- 3.13 Stopping Distances, 107
- 3.14 A Bottle with Holes, 108
- 3.15 The Pendulum, 108
- 3.16 Radio Range, 108
- 3.17 Running 400 Meters, 108
- 3.18 Blue Whale, 109
- 3.19 Used Cars, 109
- 3.20 Texts, 110

4 Nonlinear Empirical Models II 111

- 4.1 Cooling II, 112
- 4.2 Body Surface Area, 116
- 4.3 Warm-Blooded Animals, 120
- 4.4 Control of Insect Pests, 123
- 4.5 Selling Magazines for Christmas, 125
- 4.6 Tumor, 136

- 4.7 Free Fall, 141
- 4.8 Concentration, 145
- 4.9 Air Current, 150
- 4.10 Tides, 153
- 4.11 Fitness, 156
- 4.12 Life Expectancy versus Average Income, 157
- 4.13 Stockholm Center, 157
- 4.14 Workforce, 157
- 4.15 Population of Sweden, 158
- 4.16 Who Killed the Lion?, 158
- 4.17 AIDS in United States, 159
- 4.18 Thermal Comfort, 159
- 4.19 Watts and Lumen, 159
- 4.20 The Beaufort Scale, 160
- 4.21 The von Bertalanffy Growth Equation, 161

5 Modeling with Calculus

162

- 5.1 The Fish Farm II, 163
- 5.2 Titration, 169
- 5.3 The Bowl, 176
- 5.4 The Aircraft Wing, 180
- 5.5 The Gateway Arch in St. Louis, 182
- 5.6 Volume of a Pear, 187
- 5.7 Storm Flood, 190
- 5.8 Exercise, 193
- 5.9 Bicycle Reflectors, 202
- 5.10 Cardiac Output, 206
- 5.11 Medication, 210
- 5.12 New Song on Spotify, 215
- 5.13 Temperature Change, 221
- 5.14 Tar, 224
- 5.15 Bicycle Reflectors Revisited, 229
- 5.16 Gas Pressure, 229
- 5.17 Airborne Attacks, 229
- 5.18 Railroad Tracks, 230
- 5.19 Cobb–Douglas Production Functions, 230
- 5.20 Future Carbon Dioxide Emissions, 231
- 5.21 Overtaking, 232
- 5.22 Population Dynamics of India, 232
- 5.23 Drag Racing, 232
- 5.24 Super Eggs, 233
- 5.25 Measuring Sticks, 234
- 5.26 The Lecture Hall, 234

- 5.27 Progressive Braking Distances, 234
- 5.28 Cylinder in a Cone, 235

6 Using Differential Equations 236

- 6.1 Cooling III, 237
- 6.2 Moose Hunting, 241
- 6.3 The Water Container, 247
- 6.4 Skydiving, 250
- 6.5 Flu Epidemics, 256
- 6.6 USA's Population, 263
- 6.7 Predators and Prey, 274
- 6.8 Smoke, 285
- 6.9 Alcohol Consumption, 289
- 6.10 Who Killed the Mathematics Teacher, 292
- 6.11 River Clams, 297
- 6.12 Contamination, 297
- 6.13 Damped Oscillation, 297
- 6.14 The Potassium–Argon Method, 298
- 6.15 Barium, Lanthanum, and Cerium, 298
- 6.16 Iodine, 298
- 6.17 Endemic Epidemics, 299
- 6.18 War, 299
- 6.19 Farmers, Bandits, and Rulers, 299
- 6.20 Epidemics Without Immunity, 300
- 6.21 Zombie Apocalypse I, 300
- 6.22 Zombie Apocalypse II, 300

7 Geometrical Models 301

- 7.1 The Looping Pen, 302
- 7.2 Comparing Areas, 304
- 7.3 Crossing Lines, 307
- 7.4 Points in a Triangle, 310
- 7.5 Trisected Area, 316
- 7.6 Spirograph, 320
- 7.7 Connected LP Players, 326
- 7.8 Folding Paper, 332
- 7.9 The Locomotive, 336
- 7.10 Maximum Volume, 340
- 7.11 Pascal's Snail or Limaçon, 340
- 7.12 Equilateral Triangle Dissection, 341
- 7.13 Dividing the Sides of a Triangle, 341
- 7.14 The Pedal Triangle, 342

- 7.15 The Infinity Diagram, 343
- 7.16 Dissecting a Circular Segment, 344
- 7.17 Neuberg Cubic Art, 344
- 7.18 Phase Plots for Triangles, 345
- 7.19 The Joukowski Airfoil, 347

8 Discrete Models 348

- 8.1 The Cabinetmaker, 349
- 8.2 Weather, 358
- 8.3 Squirrels, 362
- 8.4 Chlorine, 365
- 8.5 The Deer Farm, 369
- 8.6 Analyzing a Number Sequence, 373
- 8.7 Inner Areas in a Square, 376
- 8.8 Inner Areas in a Triangle, 382
- 8.9 A Climate Model Based on Albedo, 387
- 8.10 Traffic Jam, 392
- 8.11 Wildfire, 399
- 8.12 A Modern Carpenter, 408
- 8.13 Conway's Game of Life, 409
- 8.14 Matrix Taxis, 409
- 8.15 The Car Park, 409
- 8.16 Selecting a Collage, 410
- 8.17 Apportionment, 410
- 8.18 Steiner Trees for Regular Polygons, 410
- 8.19 Hugs and High Fives, 411
- 8.20 Pythagorean Triples, 411
- 8.21 Credits, 412
- 8.22 The Piano, 413

9 Modeling in the Classroom 415

- 9.1 The Teacher Creating Diagrams, 416
- 9.2 Student's Lab Reports, 416
- 9.3 Making Screencast Instructions, 417
- 9.4 Demonstrations, 417
- 9.5 Students Investigating Constructions, 418
- 9.6 Working in Groups, 418
- 9.7 Students Constructing Models, 419
- 9.8 Broader Assignments, 420
- 9.9 The Same or Different Assignments, 421
- 9.10 Previous Assignments, 421
- 9.11 The Consultancy Bureau, 422

10	Assessing Modeling	425
10.1	To Evaluate Mathematical Modeling Assignments, 426	
10.2	Concretizing Grading Criteria, 426	
10.3	Evaluating Students' Work, 431	
11	Assessing Models	434
11.1	Relative Error, 435	
11.2	Correlation, 435	
11.3	Sum of Squared Errors, 436	
11.4	Simple Linear Regression, 436	
11.5	Multiple Regression Analysis, 438	
11.6	Nonlinear Regression, 438	
11.7	Confidence Intervals, 439	
11.8	2D Confidence Interval Tools, 441	
12	Interpreting Models	443
12.1	Mathematical Representations, 443	
12.2	Graphical Representations, 444	
12.3	A Sample Model Interpreted, 445	
12.4	Creating the Model, 446	
	Appendix A: Introduction to GeoGebra	448
	Appendix B: Function Library	485
	Integer Properties	509
	Index	523
	List of Problems by Name	535